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Solar Cycle Variation of the Solar Internal Rotation: Helioseismic Inversion and Dynamo Modelling

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Abstract. We report our first results on comparing the variations of the solar internal rotation with solar activity, as predicted by non-linear solar dynamo modelling, with helioseismic measurements using the SOHO MDI data.

1. Introduction

The migrating bands of faster and slower rotation of the solar surface were first discovered by Howard and LaBonte (1980). By analyzing helioseismic data which are now provided by the space (SOHO MDI) and ground-based (GONG) projects, Howe et al (2000) and Antia and Basu (2000) have found that these "torsional oscillations" penetrate quite deep into the solar interior, to at least 8 percent of solar radius.

The mechanism responsible for producing the 11-yr solar torsional oscillations is thought to be the non-linear interaction between the magnetic field and the solar differential rotation. Comparing the spatial and temporal structure of the torsional oscillations predicted by the theoretical modelling with helioseismic measurements would allow the calibration of the theoretical models of the solar dynamo, leading finally to better understanding of the basic mechanisms of solar magnetic activity.

In this contribution, we address the predictions of a two-dimensional axisymmetric mean-field dynamo model in a spherical shell, in which the only nonlinearity is the action of the azimuthal component of the Lorentz force of the dynamo-generated magnetic field on the solar angular velocity (Covas et al 2000). The torsional oscillations produced in this model are compared with the results of helioseismic inversion of the SOHO MDI data, now available over almost half of the 11-yr solar activity cycle.

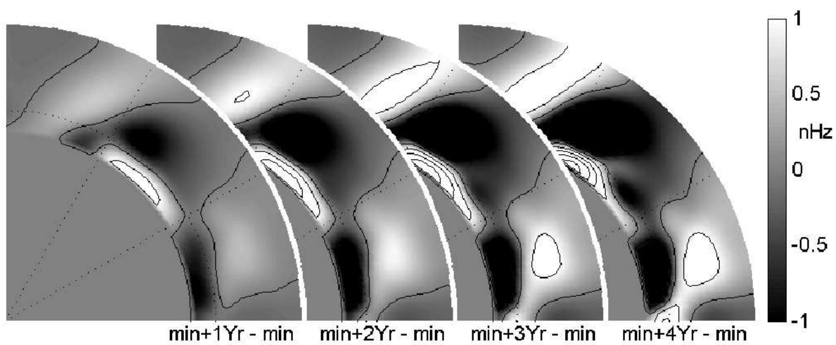

Figure 1. Variation of the solar rotation predicted by the model over one, two, three and four years of increasing solar activity. Dotted lines indicate the base of the convection zone and the 0°, 30° and 60° latitudes.

Helioseismic measurements are based on analyzing the rotational splittings of the solar p-mode frequencies. These have different sensitivities to the rotation at different depths and latitudes, which can distort the actual rotation profiles when they are inverted from the data of finite accuracy. We address this problem by using artificial inversions.

2. Torsional oscillations predicted by dynamo modelling

We consider the variations in the solar internal rotation, which are induced directly by the Lorentz force in the numerical simulations based on the axisymmetric non-linear mean-field dynamo model described in (Covas et al 2000).

The variations of the angular velocity with time, taken from the adopted dynamo model (see Tavakol et al., in preparation, for details), are shown in Fig. 1. The results are represented in a form suitable for the direct comparison with seismic measurements (see Fig. 2 below). The variations are measured relative to the solar minimum, and plotted after 1, 2, 3 and 4 years of increasing solar activity. The torsional oscillations penetrate to bottom of the convection zone, and have their largest amplitudes near its base. At lower latitudes, an accelerating "zonal flow" is clearly seen, which propagates towards the equator.

3. Helioseismic inversion

The solar data, represented by the rotational splitting coefficients inferred from the SOHO MDI measurements, and the inversion technique which we use are described in Shou (1999), Vorontsov et al. (2001), Strakhov and Vorontsov (2001).

The results of the inversion are shown in Fig. 2. The consecutive 72d data sets (Schou 1999) were averaged into one-year data sets to improve the

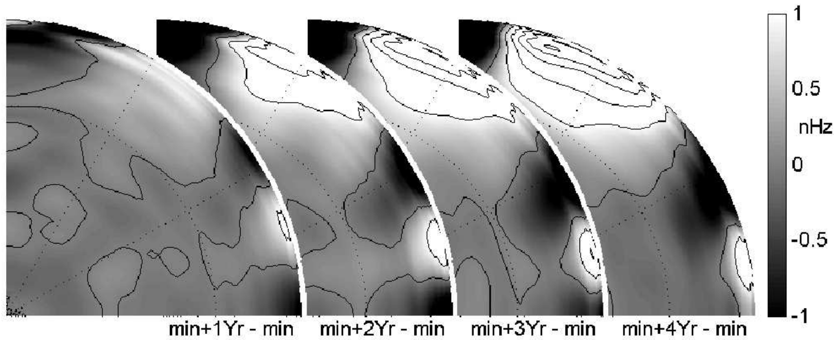

Figure 2. Variation of the solar internal rotation over one,two,three and four years of increasing solar activity, as inferred from the SOHO MDI measurements.

signal-to-noise ratio. The inversion has not been applied directly to the splitting coefficients, but to their variation relative to the first year of measurements, to reduce the effects of systematic errors.

Before comparing these results with model predictions, we note a specific feature of the adaptive regularization technique (Strakhov and Vorontsov 2001) which was used in the inversion. The regularization properties of the inversion are such that the response of the solution to the random errors in the input data (the noisy component of the solution) is nearly uniform over all the approximation domain (the meridional plane). If a particular feature in the rotation profile is well below the level of detectability (determined by data errors), it will not be seen in the solution at all; and if its amplitude is close to this level, it will appear in the solution with somewhat reduced amplitude.

4. Artificial inversion

To address the question of how the torsional oscillations predicted by the model would be "seen" in the seismic data, an artificial inversion was performed, with using the 2-D rotational profile represented by the right-hand panel of Fig. 1. The synthetic rotational splitting coefficients were calculated for this particular rotation, random errors were added, and then the result was inverted in exactly the same manner as for the real solar data.

The results are shown in Fig. 3, for inversions with different magnitudes of the added noise – those corresponding to the reported observational errors, ten times smaller, and with no errors (the error-free inversion indicates the inherent spatial resolution of the inversion technique).

The noise-free inversion works well: the only noticeable inaccuracy, near the pole, is due to the use of a truncated polynomial approximation in $\cos^2(\theta)$. With solar noise errors reduced by a factor 10, we are still able to infer the variations reasonably well, apart from their detailed behaviour near the base of the convection zone. In the inversion with solar errors, we can only see the

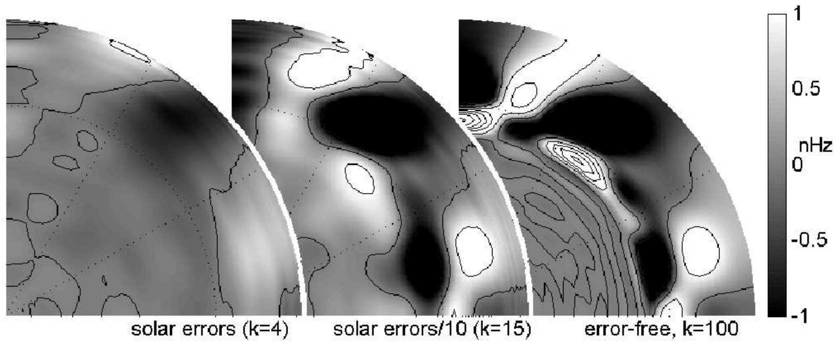

Figure 3. Artificial inversions of the variation of Ω predicted by the dynamo model over four years of increasing solar activity. From left to right: data with noise added corresponding to the reported errors of the SOHO MDI one-year data set; with noise amplitude reduced by a factor ten; the noise-free inversion.

variations in the upper half of the convection zone; the most interesting features near its base are completely buried below the noise level.

5. Discussion

When comparing the torsional oscillations predicted by the theoretical modelling with helioseismic data, we observe some general features which are in common, as well as significant differences.

Both the model and the observations show that all the convection zone, down to its base, is involved in the oscillations. Preliminary experiments suggest that this feature remains even when density stratification is included. The "zonal flows" propagate towards the equator at lower latitudes and towards the pole at higher latitudes, and the model flows show the correct phasing with solar activity.

The observed low-latitude zonal flow, however, is much more localized in depth, compared with model prediction, and closer to the surface. The high-latitude accelerating flow is much stronger than in the model, has a larger latitudinal extent, and is situated at somewhat lower latitudes.

The differences are hardly surprising; indeed, in this very first comparison we made no attempts to tune the model parameters to fit the observations, as our interest was in the effect of noise on the inversions of a known data set. These differences are also related to the inherent uncertainties in, and simplified nature of, the dynamo model: for example uniform density is assumed.

With the current accuracy of the seismic data, we are not able to resolve the well-structured variations near the base of the convection zone, predicted by the dynamo model. We deduce that such structures, if present in the 'real' Sun, also would not be reliably detected by current techniques. We believe that

the accuracy of the seismic measurements will improve significantly in the near future – due to improved accuracy in the measurement of the rotational splittings in the solar data when using the more sophisticated techniques which are being developed, and also from use of the data from ground-based observations, together with better coverage of the solar cycle.

Even with the accuracy which is now available, the seismic measurements of the torsional oscillations provide new and valuable constraints on the physical modelling of the solar dynamo. We believe that we have also shown that dynamo models can have an input into the interpretation of the seismic data.

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